TRINITY COLLEGE

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

If required by your examination administrator, please place your student identification label in this box

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (90 Marks)**

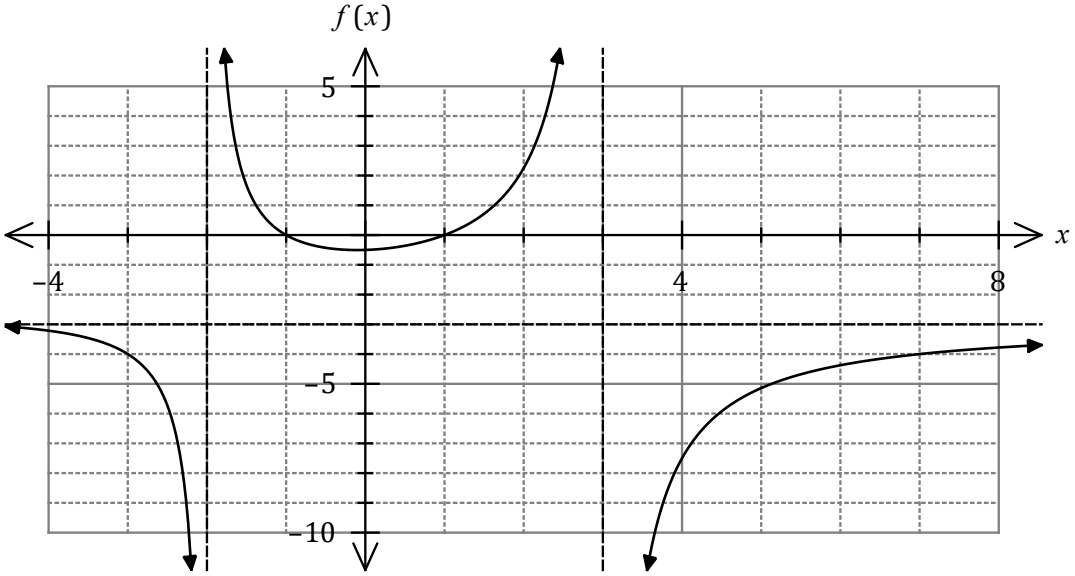
This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

The graph of $f(x) = \dfrac{a(b - x^2)}{(x - c)(x + d)}$ is shown below, where a, b, c and d are positive constants.

The dotted lines are the asymptotes of the function.



(a) Determine and write the value of each constant in the table below. (4 marks)

Constant	a	b	c	d
Value				

(b) State the equations of all asymptotes of the graph of $y = \dfrac{1}{f(x)}$. (2 marks)

Question 10 **(7 marks)**

The arguments of the non-zero complex numbers u and v are θ and ϕ respectively, and the modulus of u is twice the modulus of v.

Express the following in simplest form.

(a) $|u \div v|$ (1 mark)

(b) $\arg(iu) + \arg(\bar{u})$ (2 marks)

(c) $\dfrac{v\bar{v}}{|iu|}$ (2 marks)

(d) $\arg\left(\dfrac{\overline{uv}}{3u^2}\right)$ (2 marks)

See next page

Question 11 **(5 marks)**

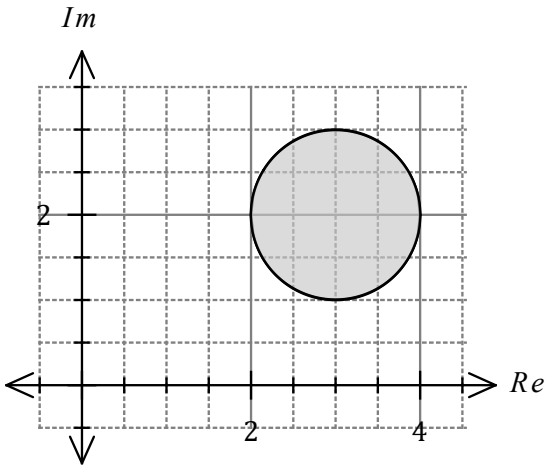
The velocity vector of a particle at time t seconds is given by $\mathbf{v}(t) = \begin{pmatrix} 2t-8 \\ 5 \\ 3e^{0.5t} \end{pmatrix}$ metres. The initial position vector of the particle is $18\mathbf{i} + 10\mathbf{j} + 2\mathbf{k}$.

(a) Determine the displacement vector $\mathbf{r}(t)$ for the particle after t seconds. (3 marks)

(b) Determine the minimum distance of the particle from the y-z plane. (2 marks)

Question 12 **(8 marks)**

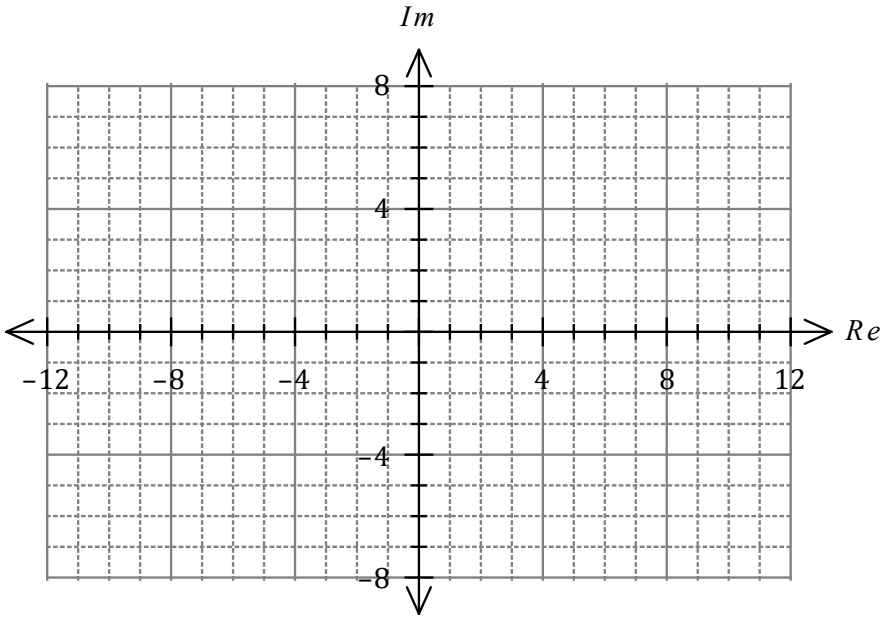
(a) The locus of a complex number z is the circular region shown below.



(i) Write equations or inequalities in terms of z (without using $\mathrm{Re}(z)$ or $\mathrm{Im}(z)$) for the indicated locus. (3 marks)

(ii) Determine the minimum value for $|z - 3i|$ as an exact value. (2 marks)

(b) On the complex plane below sketch the locus of the complex number z determined by $-\frac{\pi}{4} < \arg(z + 4 - 2i) \leq \frac{\pi}{4}$. (3 marks)



Question 13 **(7 marks)**

Functions f and g are defined as $f(x) = \frac{1}{\sqrt{x+1}}$ and $g(x) = e^{x^2-1}$.

(a) State the domain of $f(x)$ and explain why f has an inverse. (2 marks)

(b) Determine the defining rule for $f^{-1}(x)$ and state its range. (2 marks)

(c) Determine the defining rule for $g\big(f(x)\big)$ and state its domain and range. (3 marks)

See next page

Question 14 **(7 marks)**

The position vectors of the points A and B are $\mathbf{r}_A = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{r}_B = \begin{pmatrix} 1 \\ -4 \\ 5 \end{pmatrix}$.

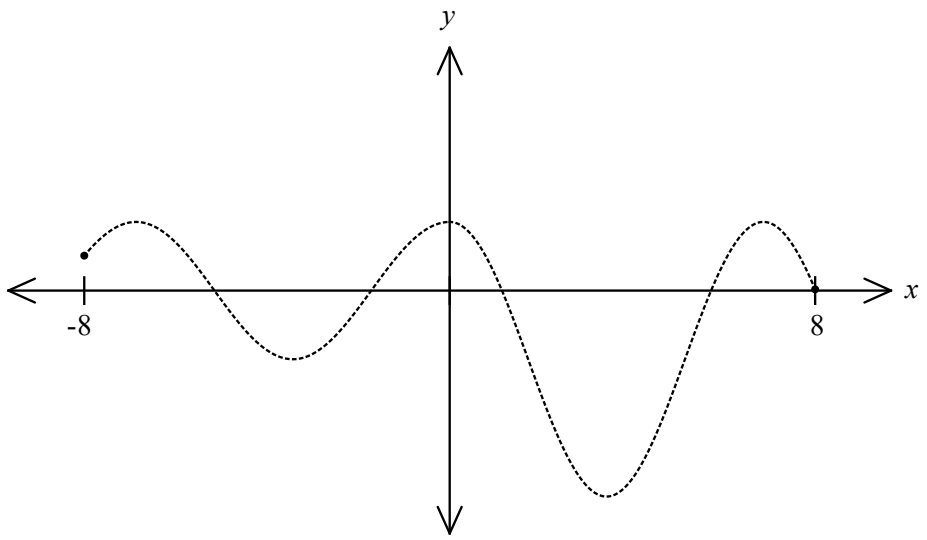
(a) If line segment AB is the diameter of sphere S, determine the vector equation of S. (3 marks)

Straight line L intersects the surface of sphere S at point A and has equation $\mathbf{r} = \mathbf{r}_A + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

(b) Determine the position vector of C, the other point of intersection of L with S. (4 marks)

Question 15 **(7 marks)**

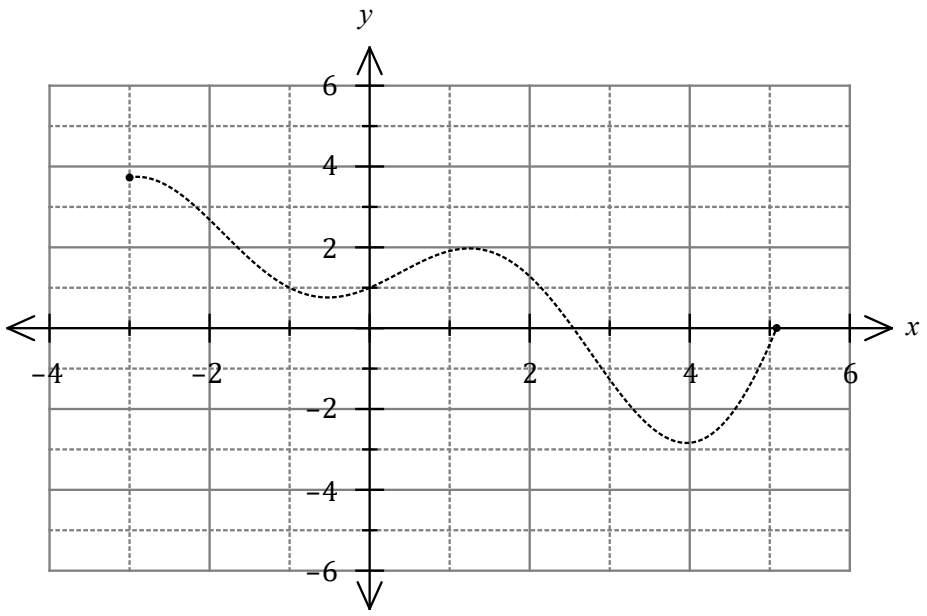
(a) The graph of $y = f(x)$ is shown with a dotted line on the axes below.



(i) On the same axes, sketch the graph of $y = |f(x)|$. (2 marks)

(ii) State the number of roots that the graph $y = f(|x|)$ will have. (1 mark)

(b) The graph of $y = g(x)$ is shown with a dotted line on the axes below. Sketch the graph of $y = \frac{1}{g(x)}$ on the same axes. (4 marks)



See next page

Question 16 **(8 marks)**

(a) One solution to the equation $z^3 = u$ is $z = 2\,\text{cis}(-40°)$.

(i) Determine the other two solutions, giving solutions in the form $r\,\text{cis}\,\theta$, where $r \geq 0$ and $-180° < \theta \leq 180°$. (2 marks)

(ii) Determine u, giving your answer in the form $a + bi$. (2 marks)

(b) Solve the equation $z^5 = 16\sqrt{2} - 16\sqrt{2}i$, giving exact solutions in the form $r\,\text{cis}\,\theta$, where $r \geq 0$ and $-\pi < \theta \leq \pi$. (4 marks)

Question 17 **(8 marks)**

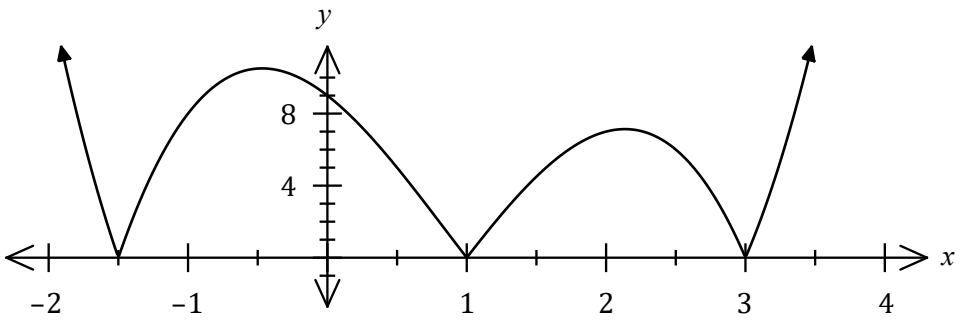
Four points in space have coordinates $A(5, 4, -5), B(6, 3, -2), C(0, -1, 5)$ and $D(-3, 0, 1)$.

(a) Show that the lines AC and BD intersect and determine the coordinates of their point of intersection. (5 marks)

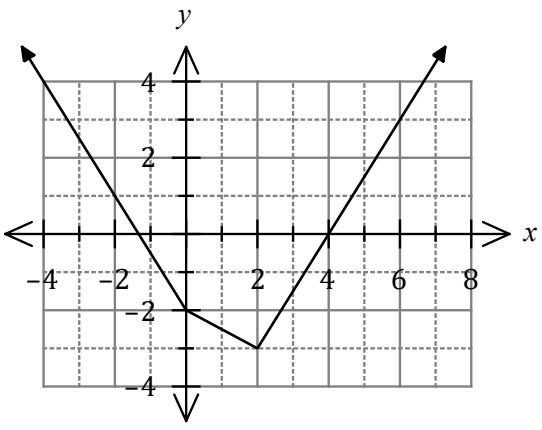
(b) Determine the Cartesian equation of the plane containing the four points. (3 marks)

Question 18 **(6 marks)**

(a) The graph of $y = |f(x)|$ is shown below, where $f(x) = ax^3 + bx^2 + cx - 9$. Determine the value of each of the coefficients a, b and c. (3 marks)



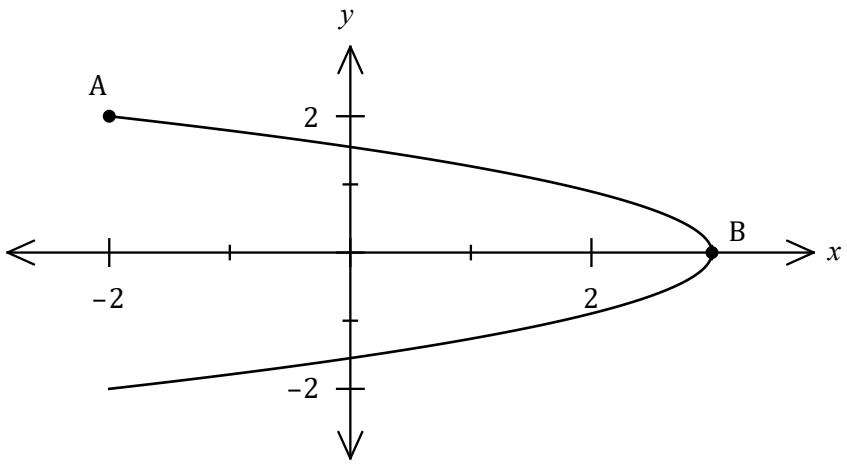
(b) The graph of $y = |px| + |x + q| + r$ is shown below, where p, q and r are constants. Determine the possible value(s) of each constant. (3 marks)



Question 19 **(8 marks)**

The position vector of a particle at time t seconds is given by $\mathbf{r}(t) = \begin{pmatrix} 5\sin^2(t) - 2 \\ 2\cos t \end{pmatrix}$ cm.

The path of the particle is shown below, together with the points $A(-2, 2)$ and $B(3, 0)$ that lie on its path.



(a) Express the path of the particle as a Cartesian equation. (3 marks)

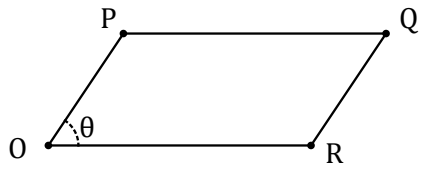
(b) Determine the velocity of the particle when $t = \frac{\pi}{4}$. (2 marks)

(c) Determine the distance travelled by the particle as it moves from A to B. (3 marks)

See next page

Question 20 **(6 marks)**

In the parallelogram shown, $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OR} = \mathbf{r}$ and the angle between the directions of $\mathbf{p}$ and $\mathbf{r}$ is θ.



It can be shown that $|\mathbf{p} \times \mathbf{r}| = |\mathbf{p}||\mathbf{r}| \sin\theta$.

(a) Explain why evaluating $|\mathbf{p} \times \mathbf{r}|$ will result in the area of the parallelogram. (2 marks)

The area of $OPQR$ is $3\sqrt{5}$ cm^2 when the position vectors of O, P and Q are $\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}a\\-2\\1\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\-1\end{pmatrix}$ respectively, with units in centimetres.

(b) Determine the value(s) of the constant a. (4 marks)

Question 21 **(7 marks)**

Let the complex number $z = \sqrt{3} + i$ and the function f be defind as $f(n) = (z)^n - (\bar{z})^n, n \in \mathbb{Z}$.

(a) Determine the modulus and argument of $f(-1)$. (2 marks)

(b) Use De Moivre's theorem to determine all values of n for which $f(n) = 0$. (5 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____